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ABSTRACT

This booklet contains 51 abstracts of papers presented at the 1992 conference for the Research Council for Diagnostic and Prescriptive Mathematics (RCDPM). Topics covered include: the use of expressive writing to enhance metacognition, adult assessment, cooperative learning assessment, visualization in problem solving, deaf students' beliefs about mathematics, using graphing calculators in precalculus, using number sentences to assess thinking, putting DPMath principles into practice, reduced symbol manipulation in calculus, assessment of language minority students, area concept acquisition, a new constructivist theory for diagnostic and prescriptive mathematics, math clinic students, hands-on geometric measuring, teaching number sense, writing to learn mathematics, using inquiry cycle methodology, effects of consistent instruction in calculus, underaddressed competencies needed by college students, study strategies for college mathematics, memorization, a methods course that considers misconceptions in secondary school mathematics, learners' theoristic development, ragged decimals, using mathematics journals, deaf education teachers' attitudes toward teaching mathematics, evolution of a classroom assessment model, assessing language skill, open-ended questions, interviewing as a final exam, van Hiele levels, math teachers as researchers, prerequisite vocabulary terms for intermediate algebra, using writing with preservice teachers, using Logo to teach variables, Mathematics Intensive Summer Session (MISS) for young women, conceptions of pictographs and bar graphs, decline of blacks in mathematics, role of language in construction of symmetry knowledge, renewal of elementary mathematics, connections, forum for prospective authors for Focus on Learning Problems in Mathematics, Diagnostic Prescriptive Workshop, school/university partnerships to integrate technology, whole language approach to mathematics, using life histories for assessing attitudes, teaching non-mathematically oriented students, Project UP with Mathematics, stimulating mathematical thinking, developmental logic, and fostering professional growth. (MKR)

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RCDPM

1992 CONFERENCE

BOOK OF ABSTRACTS

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The Use of Expressive Writing to Enhance Metacognition

by Barbara R. Allen

As a result of nationwide concern that students are not receiving the mathematics education necessary for advanced study or successful career training, educational psychologists are renewing an emphasis on how students learn. These findings support the emergence of pedagogy directed toward developing metacognition skills.

Studies of the expert-novice support the presence of metacognitive skills for the "expert" learner. Simply stated, a student must not only know what to do, but when to do it. Current literature focuses on methodology which develops skills such as reflection, self-awareness, and self-regulation. Since writing is believed to parallel powerful learning strategies, it's use may enhance students' development of metacognitive skills.

The research questions in the study to be presented will address student ability to assess the correctness of answers to test items and the effects of in-class expressive writing upon assessment ability and upon achievement.

The measurement of student ability to correctly assess solutions will be a result of numerically coding the student's indication of "very certain," "fairly certain," and "not certain" with his/her actual work on quiz items. The treatment effect on achievement and assessment ability will be statistically analyzed by using an analysis of covariance using the achievement and assessment ability scores on the pretest as covariants.

The students in this study are enrolled in an elementary algebra course taught in a four-year regional university. This non-credit course prepares the student for intermediate algebra and, subsequently, college algebra. The background of these students are varied in preparation (one - four years of college preparatory mathematics in high school), ACT math scores (1-17), and ACT English scores (1-29).

The findings and the resulting implementations will be discussed. The data was collected this spring and evaluation is currently being written.

The Search or Adult Assessment Procedures

Beatrice C. Babbitt
Virginia E. Usnick

During the past year we have been involved in the process of developing a mathematics clinic at our university. While the clientele of the clinic will eventually be kindergarten children through adults, at the present time we are focusing on university students. It seems fairly easy to identify two populations which would take advantage of such a clinic: general studies students who wish to improve their skills in mathematics and education students who have difficulty with the mathematics courses required of preservice teachers.

In order to provide appropriate services, students' individual needs must be diagnosed. Traditional assessment procedures, including those designed specifically for adults, tend to focus on computation and fail to probe logical thinking and problem solving strategies. In an attempt to broaden our pool of assessment instruments, we contacted colleagues through the RCDPM clinical network to request help in identifying more appropriate diagnostic instruments to be used in our clinic.

This session will present the results of our effort. We will compare and contrast the tests and procedures identified through our search. Sample items from selected tests will be discussed. We will identify mathematical content or processes for which appropriate adult assessments seem to be lacking. An annotated bibliography of identified tests and procedures will be provided.

Clinical Assessment of General Mathematics Students Involved in Cooperative Learning

Sandra Bethell

Mathematics curricula are still rigidly tracked in most high schools. This tracking typically limits access to powerful mathematical ideas and is generally based on a student's computational performance. The General Mathematics Project at the Holt Professional Development School has sought to provide access to the process of mathematics through providing learning activities where students can construct meaning in a learning community.

The curricula we have developed confronts students' ideas about what it means to be a learner and what it means to learn and do mathematics. A history of individualized lessons which drill computational skills and a culture of anti-intellectualism leads to fragmented ideas and a lack of social cohesion. So trying to build a learning community has been very difficult.

A collection of student work and a series of clinical interviews has illuminated many factors which impede student success. We are seriously questioning the appropriateness of the general mathematics context.

This workshop session will raise many issues regarding learning context and assessment procedures for students who have been disenfranchised from powerful mathematical knowledge. We will also describe the techniques we used to collect and analyze data.

Visualization Techniques in Problem Solving

Sue Brown and Virginia Usnick

This presentation describes a research investigation in visualization techniques used by elementary education majors and fifth grade students. The students were presented a series of problem solving questions and were asked not only to solve the questions, but also to respond to a questionnaire on visualization techniques. One question used in the study was: "Peggy got on an elevator. She went down 5 floors, up 6 floors, and down 7 floors. She was then on the second floor. Where did she get on?" After solving the problem students were asked to respond to a series of questions.

1. When you solved this problem did you have an image or mental picture in your mind?
2. If you, describe the image or mental picture? Did this help you solve the problem, why or why not?
3. If no, can you think of an image or mental picture for this problem. Describe the mental picture. Could this picture have helped you solve the problem? Why or why not?

When fifth graders solved this problem 50% of the children had an image or mental picture in their mind, while 50% of the students did not. Ninety percent of the students that formed an image or mental picture were helped by the visualization. Typical comments were: "I pictured a girl going up and down in the elevator. It helped me, because it made me think of adding and subtracting on a number line." "I thought of a little girl on an elevator going up and down. Then I 'resigned' the picture. It did help me because I knew about where she got on by 'rewinding.' For 10% of the students the mental picture did not help solve the problem. "It didn't help me and I don't know why not." The students that did not form a mental image stated: "I just used numbers." "I just reversed up and down to do it."

When elementary education majors were presented with this problem 90% of the students formed a mental picture. The students who did not form a mental picture and the students who did not find the mental picture helpful in solving the problem generally were not successful problem solvers. This presentation will provide suggestions for teachers who expect students to use visualization techniques in problem solving.

Deaf Students' Beliefs About Mathematics

Jerry Bryant

This summer fifteen hearing impaired high school students will participate in a two week course in problem solving during summer school at the Virginia School for the Deaf. One of the goals of the course is to externalize and implicitly challenge participants' traditional perceptions of and attitudes toward the study of mathematics. The proposed presentation will report results and issues arising from these preliminary efforts to examine deaf students' beliefs about mathematics.

An initial assessment of beliefs will be given by way of video taped class discussion. During the ensuing class meetings students will deal with a number of non-routine problems selected to be relatively language free and to require mathematics no more advanced than basic arithmetic. Problem solving activities will emphasize collaboration, communication, heuristics, and Polya's phases.

A post assessment will assume the form of a course evaluation. Questions similar to those on the initial assessment will be discussed as a class and video taped, but the focus will be on the recent experiences in the problem solving course.

Using Graphing Calculators to Monitor Students' Understanding of Functions in PreCalculus

Janet Caldwell

Graphing calculators provide an opportunity for students to link symbolic and visual representations, as well as an opportunity for teachers to discover more about what their students are thinking. A central idea is the precalculus curriculum is one of the concepts that frequently poses considerable difficulty for students: functions. This workshop will summarize the research relating to the use of graphing calculators, describing the effects of applying the technology in the college precalculus classroom. The presentation will also include discussion of ways in which teachers can use the graphing calculators to better diagnose student understanding of functions and provide appropriate instructions.

Number Sentences: A Tool for Assessing Children's Thinking

Deborah A. Carey

This session is designed to provide educational practitioners with information about primary school children's use of symbols, particularly how the use of symbols can be an assessment of children's mathematical thinking involving the part-whole relationship of number. Research on addition and substraction word problems suggests that problem structure influences children's solutions to word problems. The strategies that children use to solve problems become more flexible as they develop an understanding of the part-whole relationship. In a similar manner, the number sentences that children write for different work problems also reflect their thinking about the relationship of numbers in a problem. Results from a recent study on children's use of symbolic representation will be discussed and workshop participants will analyze the number sentences that first-grade children write and select for a variety of addition and substraction word problems

Putting DPMath Principles Into Practice

Doris E. Creswell

Another channel for communicating DPM developments opened in 1990 with the decision by 15 teachers to meet periodically to share ideas, problems, solutions and activities. They all have completed at least two DPM graduate-level courses and are operating DPM instructional programs. These two-hour meetings, held about six weeks apart at Penn State Harrisburg, Capital Area Intermediate Unit or local districts, often include one or two DPM university instructors as an added resource.

This special interest group's initial major project is to use members' experience in establishing and operating DPM programs to develop a manual for teachers who need help in translating theoretical knowledge into practical classroom programs. The manual will include step-by-step procedures for implementing every phase of the activity, from curriculum and setting to public relations and budget.

The group's other goals include: (1) developing and revising presentation outlines for workshops and seminars for teachers, administrators, school board members and parents; (2) developing audio-visual materials for such presentations and (3) sharing instructional and evaluation practices and materials.

The Effects on Achievement of Reduced Hand-Generated Symbolic Manipulation in Undergraduate Calculus

Robert F. Cunningham

Tradition assumes that repeated hand-generated symbolic manipulation by students is necessary for learning in calculus. Students are required to practice paper and pencil manipulations in the interest of eventually developing a deeper understanding of fundamental calculus concepts. Also, students deprived of this activity will suffer both the loss of manipulative skills and poor comprehension of fundamental calculus concepts.

Another view holds that students burdened with the learning of symbolic manipulation fail to develop an understanding of calculus concepts. So much of their cognitive energies are devoted to learning symbolic manipulation that no real conceptual understanding develops. A reduction emphasis of hand-generated symbolic manipulation can lead to greater conceptual understanding of fundamental calculus concepts.

In an effort to evaluate the role that symbolic manipulation plays in student understanding of calculus concepts, a one and a half year study was conducted. A commercially available software package capable of symbolic differentiation and integration, entitled CALCULUS written by John Kemeny and published by True BASIC Inc., was relied on by students and instructors to reduce hand-generated symbolic manipulation. The effects of reduced hand-generated symbolic manipulation on student achievement, as measured by an instrument designed to test both manipulative skills and conceptual understanding, will be reported upon. Suggestions for improved pedagogy and classroom computer use will also be presented.

Assessing the Mathematics Understanding of Language Minority Students via Interviews

David M. Davison

It is well documented that American Indian students record decreased mathematics achievement after the fourth grade. One reason suggested for this situation is inadequate attention to activities bridging the concrete with the abstract. Of particular relevance in this regard is the significance of language activities in enhancing mathematics learning. Having students discuss what they are doing as they use manipulatives and then write in English sentences what they have learned has proved to be efficacious. Further, this use of language is an effective diagnostic tool when trying to determine where there has been a learning breakdown.

This proposal addresses work in progress in three classrooms in the Billings area, including one on the Crow Indian reservation. The composition of one class is entirely American Indian, while the other two are primarily American Indian and Mexican American. For these students, it appears that mathematics achievement is affected by weak English language functioning. One purpose of the current work is to assess the students' understanding of mathematical ideas through individual interview. These interviews involve the use of manipulative and diagrams, and students explain how to perform operations with reference to the physical/visual stimuli. Preliminary data indicates that student are not able to adequately to explain the mathematical ideas which form the basis of calculations they correctly perform. This approach to assessment can be used to provide recommendations for redeveloping mathematics instruction for language minority students.

Assessment of Area Concept Acquisition Through Learning Logs

Sandra Paradis and Charles H. Dietz

Eighteen to twenty year old deaf students were asked to write about their understanding of area and perimeter five different times during the school year. The presentation will trace students' changing responses as potential indicators of their growth of understanding of these concepts. Suggestions for the use of learning logs as assessment devices of students' understanding will be included.

A New Constructivist Theory for Diagnostic and Prescriptive Mathematics

Jean D. Dionne

Mathematics constitutes a discipline where, in Quebec as anywhere else, a great many students meet with difficulties. Hence, the presence in the schools of specialists called orthopedagogues whose main function with respect to mathematics pertains to diagnosis and remedial work: that is, to determine the nature and the causes of the difficulties encountered by students who are referred to them and take appropriate measures towards solving them. Too often, their intervention is limited to the rather superficial expression of these difficulties, namely to the overtly lacking performance in arithmetical operations. Unfortunately, this aspect of learning per se does not suffice to solve the real problems: the classical example of Benny reported by Erlwanger (1973) is very convincing in this regard. This pupil succeeded in obtaining scores of 80% or more for many years without understanding the mathematics involved in the procedures that he could otherwise apply.

The limited nature of most orthopedagogical interventions can be explained by the non-existence of an explicit theory which would enable orthopedagogues to surpass the mere algorithmic aspect of mathematics. By orthopedagogical theory, we mean a referential framework in which procedures can be located in the underlying conceptual schemata in order to explain the difficulties which may occur. For instance, a pupil might not be able to count from a given term in addition simply because he does not yet conserve the number.

Our first objective therefore is to elaborate such an orthopedagogical theory. Herscovics and Bergeron's model of understanding (1988) provides us with a promising starting point for this part of our project insofar as it allows us to put forth hypotheses related to the probable causes of the observed difficulties. One could not deny the importance of continuity in conceptual schemata construction which manifests itself as much within a given schema as between various schemata. Accordingly, some difficulties could be described in terms of cognitive holes (absence of knowledge units) or cognitive discontinuities (absence of links between knowledge units otherwise present). For example, Ginsburg (1977) makes mention of many pupils incapable of establishing links between every day informal mathematics and school mathematics, thus creating a discontinuity problem between logico-physical abstraction and logico-mathematical abstraction.

It is this theory that we intend to expose: we believe that it could help orthopedagogues and other learning disabilities specialists to surpass the too limited nature of their students' know-how and really help them construct conceptual schemata that would endow their knowledge with meaning and relevance.

The second objective of our research is to test this theory. We will experiment with learning disabled students : using clinical interviews, we will study their cognitive schemata to identify, if any, cognitive holes or discontinuities, then, to determine whether we can efficiently intervene from these identifications. We will also experiment with orthopedagogues to verify if they can be initiated to the theory and use this new knowledge to diagnose and carry out remedial activities which aim at the essential, that is, the comprehension of their students.

Math Clinic Students After Ten Years

C. J. Dockweiler and Carol Simmons

Texas A&M University has sponsored a Math Clinic since 1981. During the ensuing years over 100 students from grades three through early high school have been served. This follow-up study reports on the impact of the Math Clinic on these students.

The Math Clinic routine included individual pre- and post-testing on mathematical content understanding and on the attitudes of the students toward the study of mathematics. A detailed prescription was written on each student and remediation activities were provided based on the diagnostic prescription. As expected, considerable growth was experienced during the Clinic session. Growth in content understanding as well as an improvement in the attitude of the student toward the tasks.

The follow-up study included personal contact with the students and their families in addition to the data gathered at the time of the Math Clinic and student grades and test results before and after the Math Clinic experience. The results, in summary, indicate:

- the Math Clinic is a 'worthwhile program.'
- the individual attention the students received in the Math Clinic served them well in preparing them for their later mathematics experiences.
- the learning strategies and materials utilized in the Math Clinic provided a conceptual basis which these students had not received in previous experiences.
- the dramatic gain observed during the Math Clinic experience faded soon after the sessions, but some indicators suggest a residual gain in conceptual understanding and positive attitude.

Hands on Direct Geometric Measuring

John J. Edgell, Jr.

Measuring pervades the everyday mathematical experiences of the public and is perhaps at the heart of the roots of a society plagued with poor quantative abilities. Geometric measuring is a part of the public school domain and other measuring experiences are usually built upon these experiences. Usually, students are taught a multitude of formulas which tend to lead one to believe that we are really teaching formula memorization with lots of substitution experiences preliminary to elementary algebra. Usually, students are furnished with the data for these meaningless formulas and forfeit direct measuring experiences or a real understanding of geometric or later, other measuring concepts. This workshop/presentation/paper will present three (3) years of field/experiences in teaching fifth grade students ("gifted/talented") direct geometric measuring in 1-D, 2-D, and 3-D spaces using one basic notion of tessalation and one related formula in each of the spaces. Basically, the fundamental formulas of Pythagoras, Heron, and Carnot will be explored and related, leading to the conjecture (Eve's conjecture) that there is one general formula for measuring n-D space.

Teaching for Number Sense Now! - Reaching the NCTM Standards

Francis (Skip) Fennell

Need

Is 17 closer to 10 or 20?
Is 39% closer to 1/3 or 1/2?
Place the decimal point where you think it should be for: 15.24 x 4.5 = 6858
The height of a 10 year old boy is 5 feet. What do you think his height will be when he is 20?

Number Sense. That's what the examples above are all about. Students need to understand what happens with numbers, and teachers must feel comfortable teaching such concepts. In order for students to develop flexibility in expressing numbers, they must see numbers in a variety of contexts and situations. Sometimes this will be an exact response, sometimes not. "The major objective of elementary school mathematics should be to develop number sense "Everybody Counts, 1989, p. 46. Number sense is important for all of us. It's the ability to use number concepts with the least amount of effort. Students who have number sense, understand numbers and know how and when to use them. Students with number sense have multiple meanings of numbers, operations and the use of the procedural aspects of early mathematics learning activities. The elementary student of the 1990's must have number sense. It is the basic skill of the decade. Knowledgeable teachers are the key to insuring that number sense becomes a high priority in all classrooms.

Plan

Number Sense Now! will produce a set of three videotapes and accompanying print support materials, to demonstrate and promote number sense, utilizing approaches consistent with and supporting the National Council of Teachers of Mathematics' (NCTM) ***Curriculum and Evaluation Standards for School Mathematics.*** The project videotapes will be designed for elementary school teachers and will involve classroom teachers actively engaged in number sense activities with children.

The purpose of **Number Sense Now!** is to inform the classroom teacher of the importance of number sense as a critical element in mathematical communication. This project will show teachers how number sense can be an integral component of their daily mathematics teaching. Number Sense Now! will involve NCTM, the NCTM Task Force on Number Sense, Western Maryland College, the Baltimore, Carroll and Howard County Public Schools (Md.), and the Washington, DC Public Schools.

Significance and Intended Outcome

The impact of Number Sense Now! is in the area of teacher and curriculum improvement. This project has national implications because, prior to the release and discussion of the NCTM ***Curriculum and Evaluation Standards for School Mathematics***, number sense was not included in elementary teacher preparation programs. The intent of the project is to encourage teachers to use the number sense instructional activities and strategies exemplified in the NCTM Standards. Project teachers, selected from the participating school districts, will help create lessons which involve number sense and accompanying print support materials. There will be 6-7 project teachers at each of three instructional levels (grades 1-2; 2-4; and 4-5). Completed videotapes and print materials will be provided (gratis) to each state mathematics supervisor. Additional tapes will be made available through NCTM.

Writing to learn mathematics: Enhancement of Mathematical Understanding

Aparna B. Ganguli

Many entering college students face severe difficulty in mathematics because of their lack of understanding in mathematical concepts. Most generally, in their history of learning mathematics, they were asked to work out the problems by manipulating numbers only. In a mathematics class they were seldom given the opportunity to express their thoughts and consequently clarify their understanding in mathematics. One important component of learning mathematics 'how to think' was never exercised for these students. This presentation will focus on how writing assignments helped remedial students to gain insight into mathematical understanding.

The purpose of this study was to examine the effect of regular 'writing to learn mathematics' activities on the development of students' mathematical understanding. Twenty-five college students enrolled in an intermediate algebra class participated in the study. Students were not given any writing assignments during the first twenty lectures of the quarter. In each lecture, during the last ten lectures of the quarter, the same students were given one five to seven minutes in-class writing assignment. The writings were collected, graded on a ten-point scale and returned to the students with appropriate comments. Students' understanding of algebra before and after the integration of writing assignments were compared.

The results indicated that students started to make sense out of the mathematical terms after integration of writing. For example, many students for the first time understood the proper meaning of the term coefficient, exponent, linear equation etc. as they wrote. Some writing assignments were developed for the purpose of brainstorming. For example, students were introduced to the concept of rational expressions but the study of the graphs of rational functions were beyond the scope of the course. A writing assignment included a graph of a rational function and called for writing a story about the graph.

An analysis of student writings indicated that, although the concepts of intercepts, slopes, excluded value of a rational expression etc, were discussed at length in class and students did many homework problems on these, majority of the students did not internalize these concepts. Students writing, in many cases, proved that students failed to think mathematically. Only one student (4%) wrote about the value of x where the rational expression could be undefined while another (4%) student wrote about y-intercept where the numerator must be zero. A discussion after the graded assignments were returned indicated that students never learned to interpret, assimilate or connect previously learned knowledge to the present structure. But after completing five activities of writing, consulting and then rewriting the majority of the students demonstrated an enhancement of mathematical understanding as evidenced form the analysis of their end of the quarter writing assignments.

A Study of the Validity and Utility of Brief Inquiry Projects as Methods for Simultaneous Assessment of Mathematics and Science Concepts, Principles, Problem Solving, Inquiry, and Communication

Jayne Fleener, Susan Westbrook & Laura Rogers

The University of Oklahoma is involved in a project which uses inquiry cycle methodology to move firmly in the direction of complex assessment and multiple indicators of process as well as outcome. Traditional classroom assessment procedures involve a trade-off between ease of gathering evaluative data and artificiality of the performance assessed. This artificiality will become even more pronounced as mathematics instruction is re-shaped by the recent NCTM *Standards* and comes to resemble more the hands-on, inquiry-oriented activities typical of better science instruction. A second problem beyond the ease-artificiality trade-off is that of the isolation of mathematics and science assessment. Traditional assessment practice embodies a rigid distinction between science and mathematics instruction and testing. A third major problem is that traditional assessment is likely to be outcome-oriented, to be focused on performance by individuals, and to take place in an artificially constrained communication environment of brief questions and stereotypic answers. This sort of assessment becomes increasingly more problematic and less valid a instruction moves towards assessment of process as well as of outcomes, makes use of such strategies as cooperative learning, and moves towards a greater emphasis on substantive, natural-language communication. A re-thinking of tools and strategies for assessment of higher order thinking and reasoning skills has suggested the need for multiple assessment indicators or profiles.

This short report will focus on efforts to (1) develop brief inquiry projects in mathematics and science instruction that will provide less artificial arenas for more complex, richer assessment; (2) use such inquiry projects to make simultaneous assessment in mathematics, science, learning and inquiry skills, and communication proficiencies; (3) develop simple methods for on-the-fly gathering of evaluative data and impressions by classroom teachers to be summarized in a practical, multiple-area "scorecard" that provides a more in-dept profile of general student thinking and communication skills as well as those peculiar to mathematics and science; (4) evaluate the utility of this assessment procedure in actual classrooms by practicing elementary and middle school teachers trained in its use; and (5) evaluate the validity of assessments provided by the procedures developed by comparing the valuative portraits achieved with those achieved by other, appropriately comparable assessment procedures: paper and pencil objective and short answer tests, open-ended brief "essay" or problem-solving tests, and clinical interview assessment methodology.

The Effect of Consistent Instruction on Mathematical Achievement in Calculus

Susan L. Ganter

During the last five years, the calculus course as we know it has been under severe scrutiny. The bottom line is that we must increase the number of students who are successfully completing the calculus sequence. Nationwide, many projects are investigating ways to achieve this goal. But what can we do now?

This paper will explore one possible answer to this question. A pilot study which included interviews with students at various stages of the calculus sequence revealed one aspect of the course which the students viewed as a problem: The instructor. Not who the instructor was but rather how that person varied from quarter to quarter. Many students indicated that the first several weeks of any given term is primarily spent "figuring out" the instructor. They felt that for this reason, having the same instructor for the entire sequence would be very beneficial to their learning. If this is true, then we will have a relatively easy way of increasing the persistence and performance of students in calculus. The following study was developed to test this hypothesis and to assess the value of consistency in instruction in the calculus sequence.

During the 1990-1991 school year, three calculus classes at a four-year university were involved in an experiment to determine the effect that a consistent instructor would have on achievement. Three different instructors were assigned to teach the calculus sequence over the course of three quarters. The students in these courses were free to enroll with the instructor of their choosing each quarter. The following data for each student were recorded: instructor for each quarter, highest quarter of the sequence completed, final exam scores, and grades in the course for each quarter.

The instructor for each quarter was observed in order to make comparisons between students with the same instructor for each quarter (consistent instruction), as opposed to those with different instructors each quarter (inconsistent instruction). Student continuation rates were also recorded for each instructor likely to complete the full-year sequence than the other students. This data was also used to reveal any student preference for consistent instruction by looking at the number of students choosing to keep the same instructor throughout the year.

A common final exam, written by the instructors, was given in all three classes during winter and spring quarters. The scores were used to determine the success of each student as compared with the other students from their previous class. The course grades were used to compare students with the same instructor during a particular quarter as a baseline for comparing those students in the following quarter.

Preliminary results indicate that consistent instruction produced slightly better performance levels. The exam scores of students with consistent instruction tended to be higher than those of students who changed instructions. Even more promising, however, are the course completion results. The percentage of students completing the full-year sequence with the same instructor was higher than for those completing with different instructors. Therefore, the data indicates that consistent instruction could increase the number of students completing a full year of calculus.

Some Underaddressed Competencies Needed by College Students

Joe Garofalo

Most developmental (remedial) college mathematics courses are designed to help students prepare for the mathematical demands of college. However, too often these courses are designed without sufficient knowledge of the range of mathematical competencies that students need in order to be successful in courses involving mathematics. In an attempt to gain a better understanding of the mathematics students need, we surveyed a variety of courses in disciplines which use mathematics (e.g. biology, commerce, economics, environmental science) by interviewing instructors, auditing course lectures, and analyzing textbooks and exams.

Through the survey, we identified an array of mathematical competencies students are expected to have as prerequisite. In particular we identified a number of very important competencies in which many students are deficient. These include:

> Students must believe and understand that mathematics can be used to describe, model, and analyze situations arising in other disciplines, and moreover, that the mathematics used to do such is the same as, or at least is related to, the mathematics they studied in high school.
>
> Students must be able to construct, interpret, and represent functional relationships, particularly those arising in applied contexts (not just $y=f(x)$).
>
> Students must be able to solve multi-step and non-routine problems (not just problems whose types have been rehearsed)
>
> Students must be able to read and write mathematical material (concepts, methods, explanations), often in courses other than mathematics courses.

In this session we will illustrate and explore deficiencies students have with respect to these and other competencies. We will pay particular attention to deficiencies associated with functional relationships. Also, we will describe our efforts to redesign a pre-calculus course to better address needed mathematical competencies.

Study Strategies for College Mathematics

Susan N. Gilkey

Students who are able to communicate effectively possess many of the skills necessary for academic success. Taking notes during lectures, reading textbooks, integrating lecture notes with textbook notes, acquiring necessary vocabulary, writing papers, and studying for examinations are critical to academic achievement in all disciplines. Mathematics is no exception.

During Fall 1990, I served as a discussion leader in the Academic Achievement Program for undergraduates at the University of Virginia. My assignment within this program was to help students enrolled in precalculus choose appropriate study strategies for given assignments, apply and adapt the chosen strategies, and evaluate the results of using particular strategy. I worked with students to maximize their effectiveness in taking notes during class lectures, reading mathematics textbooks, annotating reading notes, mastering mathematics vocabulary, understanding algorithms and concepts, developing study sheets, and preparing for examinations.

My presentation will demonstrate the study strategies I taught and the relative strengths and weaknesses of particular strategies. I will also report on the application my students made of those strategies, including successes and failures, during our semester together and during the spring term that followed.

Is it Live or Memorized?

Deborah Gilman-Day

Mathematics and arithmetic have historically been taught and diagnosed by evaluating abstract paper and pencil computation. Little effort is given to clinically observing the thought processes and conceptual understanding which underlies the student's responses. The DPMA (Diagnostic-Prescriptive Math Assessment) is a diagnostic/ prescriptive procedure which relies on an interactive, questioning strategy using manipulatives to represent a client's understanding of mathematics and arithmetic. The student is asked to explain his/her responses in detail and questions can be paraphrased to ensure the student's understanding of clinical tasks. Because this procedure is so interactive, students often comment that is not like any other testing experience.

The authors also postulate that there are two distinct approaches to learning mathematics, a linear/algebraic approach and a geometric/spatial approach. These styles are not mutually exclusive but rather cooperatively used in diagnosis and instruction to provide a holistic, conceptually balanced approach to mathematics. Educational classroom instruction utilizes the linear/algebraic approach and assumes that the learner is able to fill in the spatial/geometric correlations. The DPMA can ascertain the personal preference of the student and can provide the examiner with an explanation and a rationale for a more balanced approach to instruction.

The DPMA has been field tested with a diverse population of learners. The age range is from kindergarten to ninth grade. The ability range has been from learning disabled/dyscalculic students to gifted populations.

It is a valuable tool for the implementation of a maximal mathematics discipline; it guides the educator in an individual, incremental approach to number, numeration, measurement and geometry and logic.

A Preservice Methods Course that Considers Misconceptions In Secondary School Mathematics

Anna O. Graeber

Participants will have an opportunity to experience some activities and examine materials from a preservice (or inservice) course for secondary mathematics education majors. Development of the course was undertaken because there was growing evidence for and acceptance of the theory that students construct their own knowledge. Those who accept this theory generally agree that students' construction of knowledge is greatly influenced by a variety of factors including the students', their peers', their parents', and their teachers' attitudes and beliefs about mathematics as well as the students' understanding and interpretation of earlier learning (See for example, Linn 1986). The course materials, developed as part of a National Science Foundation Teacher Preparation Grant, stress consideration of students' thinking, include information on many common misconceptions and provide material on some of the theoretical models for building instruction to help students overcome the influence of such misconceptions. Also included are activities that incorporate the use of manipulatives, promote consideration of textbook and software materials, and suggest techniques for teaching problem solving.

The course materials reflect the view that preservice teachers should:

--be aware of the nature of the secondary school population, the understandings they are apt to bring, and the understandings mathematics educators suggest they need after high school
--be knowledgeable about some of the specific difficulties students are apt to exhibit, understand that all difficulties cannot be known ahead of time, and be alert to the cognitive processes and curricular and pedagogical influences that seem to be the sources of such difficulties
--be aware of teaching strategies that hold promise in effectively changing students' ideas.
--have some supervised experience tutoring a high school student prior to their student teaching experience

The course materials are primarily intended for use by college and university instructors of preservice secondary mathematics teachers. However, it is clear from our work in developing these materials that inservice instructors and instructors of college developmental mathematics courses will also find the resources helpful.

The grant was funded in 1988 and materials revision will be complete in August 1991, well before the workshop presentation. Efforts are not underway to seek commerical publication. Whether via commercial publisher or through at-cost sales, materials will be available for use by other instructors.

Workshop Agenda:
Presentation on the purpose, objectives, and University of Maryland experience (10 minutes)
Participants complete some of the activities (20 minutes).
Participants peruse materials and ask individual questions (15 minutes)
Whole group question/answer session (5 minutes)

CRITICAL POINT METHODS FOR CLINICAL ASSESSMENT OF LEARNERS' THEORISTIC DEVELOPMENT

Clyde L. Greeno

By their natures, clinical instruction, teacher education, and research necessarily focus on the mathematical theories under development in the minds of the individual learner: their contents and structure and the learner's personal powers for extending and employing them. In all of those clinical contexts, clinical instruction is concerned with guiding the development of those personal mathematical theories and cultivating those powers, often to alleviate learner deficiencies.

The effectiveness of clinical prescription therefore is heavily dependent on the diagnostic acuity of the methods and instruments employed for assessing the student's state of theoristic development. Classical methods and instruments are designed from perspectives that prevail in curricular class instruction and are quite inadequate for clinical purposes. Instead, the clinical community faces the challenge need of generating a new technology of state-development assessment that is more appropriate for its own use.

This presentation airs a general methodology that derives from theoretical mathematics instructology and especially from the State-Transition Analysis and Guidance (STAG) Methods used therein. The methodology parallels that used in clinical medicine which relies, rather than on single comprehensive instruments, on a multitude of tests, each of which has a very narrow scope and very specific purposes.

The general character is that of a Mini-Test Composite System of Assessment. Its use requires a catalog of specific tests, each indexed to the particular developmental factors that it can help to examine. The compilation of a comprehensive universal catalog is a horrendous task, and one not likely to be achieved quickly. However, Critical Point Methods enable local clinics, which normally specialize in particular kinds of services for specific populations, to develop adequate in-house systems of that kind at the cost of relatively little effort, continuously applied.

STAG Methods reveal that, by invoking the known structure of established mathematical knowledge, it is possible to obtain relatively accurate profile of the learner's present state of development, in comparison to a preferred state of development, through examination of a modest number of points that are critical to achievement of the preferred state. (In medicine, such tests are called "critical signs.")

The methodology will be outlined for such use in local clinics and guidelines for its use circulated to attendees. Only example mini-test items will be considered, but the MALEI Institute will offer to coordinate community-wide network collaboration in the continuous compilation of a catalog of mini-tests.

Ragged Decimals - Teaching, Testing, and Measurement

Boyd Holtan

Teachers are using "ragged decimals" when they add or subtract decimal numerals representing small measures of different precision. Surveys of elementary mathematics textbooks indicate that the sums and differences are found by annexing zeros before the operation. If the problem represents a measurement situation, the digits which result from annexing zeros are meaningless. Teachers also have difficulty producing meaningful application situations when the solution would be the sum or difference of ragged decimals.

Teachers insist that they must continue this practice since that is the solution expected on standardized tests. Observation of some standardized tests indicate that to be true.

Some questions to be considered are:

•Should we continue to teach operations that do not correctly model world applications?

•Should we continue to teach operations that are incorrect because they are expected on standardized tests?

Can children be taught to "round to the same precision" before adding or subtracting ragged decimals.

How can students know what results on their calculators are "garbage" digits?

Using Mathematics Journals with Community College Students

Dawn Hoyt Kidd

The use of writing in the mathematics classroom has become a more prominent idea recently, especially with the ideas in the NCTM Standards concerning mathematics as communication. One way to incorporate writing into mathematics learning is the use of mathematics journals. Journals, also known as "logs" or "portfolios," are writing exercises, done by the student, whose topics are linked to mathematics instruction. As students write about their feelings, concerns, and difficulties in mathematics, the instructor gleans information about individual student learning styles and capabilities that might not be discovered otherwise. The use of journals could additionally be thought of as an informal assessment measure.

Method

Nine students enrolled in a basic mathematics course at a mid-size community college were asked to keep a mathematics journal for the duration of the semester as part of the class requirements. The class was entitled "Basic Math Skills," and covered beginning mathematics concepts including operations with whole numbers, fractions, and decimals. The class met twice a week for one hour and fifteen minutes. Journals were collected at the beginning of the week and were returned the next class period. Assignments were either left to the student to decide, or were specific questions or problems posed by the instructor; all were mathematically related, often to current topics being covered in class. The instructor read the journals and wrote pertinent feedback to the students. All journals were collected at the end of the 16-week semester from students who agreed to participate.

Journals were analyzed for entries that indicated learning had occurred, as well as for creative entries that gave the student a different perspective on a given concept. Examples from journals will be used in the presentation, and recommendations for using mathematics journals at the community college level will be discussed.

Pre-service Deaf Education Teachers' Conceptions of and Attitudes Toward Teaching Mathematics

Dawn Hoyt Kidd and Anne Madsen-Nasen

The purpose of this informal study was to examine the knowledge and attitudes of pre-service deaf education teachers concerning the teaching of mathematics. The focus of most deaf education teacher training programs is on instructional methods in the areas of speech and language. Mathematics instruction in general is not usually a major concern of deaf educators. There is little research done on the teaching and achievement of deaf students in mathematics. There is a generally accepted ideas that mathematics only involves the manipulation of numbers, therefore deaf students should do as well as hearing students despite the language difficulties of the deaf. All this may have an effect on deaf students' mathematical achievement, as it is usually lower than hearing students'.

A small group of pre-service deaf education students at a large university was asked to complete a questionnaire concerning mathematics and teaching mathematics. All students were in their final semester at the university before graduation, and were in deaf classroom placements for student teaching requirements. The questionnaire contained four sections: Conception of Mathematics, Teaching Mathematics, Classroom Procedures, and Instructional Strategies. Most of the responses were recorded as short answers or on a Likert-type scale. Analysis of the responses as well as conclusions will be discussed with examples taken from the questionnaires. Recommendations include the need for more research and attention to mathematics instruction within the realm of deaf education.

Bottom-Up Evolution of a Classroom Assessment Model: Primary Mathematics

Trish Koontz and Gen Davis

This study focused on fundamental components and behaviors teachers must internalize in order to appropriately and effectively assess children's mathematical learning. During this three year funded project 28 primary teachers participated in workshops, monthly progress meetings, team teaching with university faculty, and parent updates. Data concerning the teacher's belief systems on how children learn mathematics was collected on an ongoing basis through interviews, video tapes, journals, and monthly discussion meetings. After 15 months of the project, teachers reported being uncomfortable with the traditional assessment process. Feeling more comfortable with the constructivist approach to teaching, teachers expressed a need to develop alternative assessment processes for primary mathematics.

This presentation will outline the evolution of changing teacher's belief systems of how mathematics is learned and the development of an alternative assessment process for primary mathematics learning.

"Assessing Language Skill in the Mathematics Classroom"

Charles E. Lamb

The NCTM Standards (1989) list the ability to communicate mathematically as a key goal for school mathematics. Classroom assessment of such skills are an integral part of successful instruction.

The purpose of this session will be to discuss key ideas related to the classroom assessment of language arts proficiency related to mathematics. Methods and underlying rationales will be discussed. Assessment techniques will be both of a formal and informal nature. They will cover the broad language arts skills of listening, speaking, reading, and writing. Each of these language arts skills will be linked to the final product of mathematical thinking and reasoning.

Developing Open-Ended Mathematics Questions, and Generalized and Specific Rubrics for Rational Numbers Topics

Susan J. Lamon

The interdependence of mathematics instruction and assessment makes innovations in mathematics assessment the business of all mathematics educators. Current trends in assessment predict that the process of creating and scoring open-ended mathematics questions will continue to grow in importance as an alternative to using single-answer test questions. Not only test makers, but researchers and practitioners will be developing questions and scoring rubrics.

This session will promote the cooperative development and scoring of assessment items within small working groups. After a brief presentation by several mathematics educators involved in assessment, working groups will be formed and guided through the process of 1) deciding on the cognitive goals for a given rational number topic, 2) developing open-ended questions, and 3) developing scoring rubrics. Group processes, products, and participant input and reactions to the group negotiations will be discussed. Discussion will highlight the process by which participants were able to achieve their goal, the mathematical and pedagogical content knowledge relevant to the task, what "worked," what difficulties were encountered, what topics needed negotiation, how and in which dimensions the question evolved, and the amount and types of guidance provided by the group facilitators. Finally reactions and suggestions of facilitators and other participants will be discussed with an eye toward using this type of session to preservice and in service teachers.

Alternative Assessment in Mathematics Education: Using an Interview as a "Final Exam"

Patricia M. Lamphere

As an alternative to a final examination for a course in Teaching Mathematics in the Middle Grades, students agreed to be interviewed. Conducting interviews with thirteen students was more time consuming than the regular two-hour exam; however, I felt it would accomplish three goals. The first goal was to model for the students that alternatives do exist for the normal paper-and-pencil assessment most of them are familiar with. Second, the students' responses would demonstrate whether they had internalized the information covered in the course. Third, the process gave the students an opportunity to practice for their up-coming student-teaching interviews. Each interview lasted approximately 20 minutes.

Questions addressed during the interview included the following:
1. What guides your philosophy of teaching middle school mathematics?
2. What kinds of teaching strategies do you plan to use in your classroom?
3. What do you feel are the most important components of a middle school mathematics program and why ?
4. How will you assess what the students are learning?
5. How is teaching mathematics at the middle school level different form teaching at the high school or elementary levels?
6. Describe how you plan on conducting your daily lessons.

The students responded very favorably to the interview. One student, who has a learning disability, was very glad to have an opportunity to discuss what he knew in a form different from the traditional method. The students prepared for the interview in the same way as they would have prepared for a traditional exam except for one aspect. Most of the students worked with a friend or family member and practiced responding to a set of questions they thought might be asked during the "interview."

The grade received was based on the student's knowledge of the material and his or her general presentation. In the response to the question concerning philosophy, students either referred to a humanistic approach or a content approach. Responses to each of the questions will be analyzed to determine if any particular themes appear, such as: humanistic approaches vs. content approaches, hands-on vs. algorithmic, drill and practice vs. novel activities. Most importantly, the students realized that developing and using alternative forms of assessment is possible, even at the college/ university level.

What Can Commercial Tests Tell Us About van Hiele Levels?

Marguerite Mason

Dutch educators P. M. van Hiele and Dina van Hiele-Geldof proposed a linearly-ordered model of geometric understanding. The van Hiele theory asserts that there exist five hierarchical levels of geometric thinking that a successful learner passes through: visualization, analysis, abstraction, deduction, and rigor. The learner cannot achieve one level without passing through the previous levels. Progress from one level to the next is more dependent on educational experiences than on age or maturation, and certain types of experiences can facilitate (or impede) progress within a level and to a higher level.

Many instruments and techniques for determining a student's van Hiele level have been devised: criteria based on the number and pattern of correct answers on a written test (Usiskin, 1982), the number and pattern of correct answers on an oral test (Mayberry, 1983), the thinking levels demonstrated while performing specific activities during an oral interview (Burger & Shaughnessy, 1986; Fuys, Geddes & Tischler, 1988), and a combination of the level and the mathematical accuracy displayed on an open-ended written test (Gutierrez, Jaime, & Fortuny, 1991).

These instruments were developed primarily for research purposes and are not widely used by, or even available to, classroom teachers. There are over 20 commercial achievement and diagnostic tests available which contain a geometry component (Braswell & Dodd, 1988). This presentation will analyze the geometry portions of several of these widely used commercial tests such as the Iowa Test of Basic Skills as to the van Hiele levels required to correctly answer the individual test items and what prescriptive information these tests can provide to a teacher who wishes to structure his/her instructional activities based on the van Hiele model.

The Math Teacher as Researcher: A Change in Perspective which Identifies the Math Teacher's Psychology of learning as a Neglected Area for Mathematics Learning

Egon Mermelstein and Ingrid Thompson

A case is presented for the teacher - researcher. As researcher, the teacher is made aware of his/her psychology of learning. To promote an understanding of the teacher's psychology of learning, the narrative is the method used. Two narratives, one by a writing teacher and one by a math teacher, are examined by phemenological exegesis, that is, a line by line reading of the texts (narratives) that encourages "asking the right questions."

The findings indicate that the two teachers' psychology of learning, the exploration of error, influences the course content and the students' learning of the content.

What a mathematics teacher - researcher uncovers is that there is an interaction between his psychology of learning and the mathematical content, that is, mathematics content is not independent of the observer presenting that material. Teachers discover this important perception only by observing and recording their own development of a psychology of learning.

Assessment of Prerequisite Mathematics Vocabulary Terms for Intermediate Algebra

Cynthia A. Miller, Ph.D.

In a previous study by Miller, Smith and Grossman investigating the relationships among reading, writing, and mathematics, the researchers concluded that many students are reading at the frustration level of mathematics. As defined by Betts, this means that more than 10% of the words in a passage are unfamiliar to the students. We hypothesize that assumptions are being made about prior word knowledge that may not be true and may, in fact, result in students viewing their mathematics textbooks as a format for listing homework assignments rather than a tool for learning. In order to check this hypothesis and to compare the mathematical vocabulary of Developmental Students with regularly admitted freshmen mathematics students, we gave pre and post vocabulary tests. I will present the results of this joint project by professors in Developmental Studies Mathematics (Dr. Cynthia Miller) and Developmental Studied Reading (Dr. Brenda Smith) in which we correlated mathematics vocabulary with overall course grade, and tested for difference between Developmental Studies and regular Mathematics students on pre and post algebra vocabulary tests.

Using Writing In Mathematics Education Courses for Pre—Service Teachers

Melfried and Judith Olson

The NCTM Standards address the need for communication in mathematics. One form of communication being suggested is writing. One form of communication via writing can be labeled "learning to write about mathematics," while another form can be labeled "writing to learn about mathematics." Both forms will be discussed in the presentation.

Examples of student writing from a Mathematics Methods Class for Middle School and Junior High and from a Reasoning in School Mathematics Course for students in Elementary Education seeking an area of concentration in mathematics will be examined. In addition, ideas for using portfolios for assessment as well as for organizing of completed work will be discussed. Sample questions, sample problems, and samples of written assessment items concerning the portfolio will be provided.

Using Logo to Help Students Overcome Difficulties with Variables

Enrique Ortiz

Two of the main reasons for teachers and curriculum developers to carefully analyze the effectiveness of different approaches in teaching the concept of variable are as follows. First, it is one of the most important themes in mathematics which serves to organize, unify and give coherence to mathematics at all levels. This is affirm when the Curriculum an Evaluation Standards for School Mathematics (NCTM, 1989) includes it as one of the most important concepts for all grades. Second, this concept presents some difficulties for the students. This has been documented by several researchers.

Logo is one of the approaches which provides a unique conceptual framework that can enhance the learning of variables. Teachers' skills in connecting work with Logo and standard mathematics is crucial. They should be able to assess students' difficulties with variables and use suitable approaches based on this information. Some students have a difficult time:

1. Visualizing variables as representing just one value (an unknown) rather than a range of values (a dynamic view of the concept).
2. visualizing two or more variable representing a range of values in an expression or equation,
3. understanding that, whereas in ordinary algebra, X = X + 2 suggests an equation with no solution, in computer science the same sentence is acceptable,
4. thinking that changing the literal symbol in an equation may also change the solution of the equation,
5. assuming that two variables being different letters could represent the same number
6. using zero as a possible value for a variable, and
7. failing to describe the degree to which changes in one set relate to changes in another

The discussion will include methods for assessing students' difficulties with concept and how Logo could be used to help students avoid or overcome these difficulties. No previous work with Logo is necessary.

Mathematics Intensive Summer Session

David Pagni

The Mathematics Intensive Summer Session (MISS) engages young women high school students in the study of mathematics in an environment that helps them: 1) discover and rectify weaknesses, and 2) realize their potential by challenging them with new mathematical content. MISS students studied mathematics 6 hours a day (8 am -3 pm) for one month (July 9-August 3, 1990). Through small group learning, and a small teacher to student ratio, students showed gains in mathematics achievement, mathematics self-concept and mathematics attitude. Students were also empowered through the use of technology in the form of a graphing calculator. Follow-up of students in their senior year revealed that they were succeeding in Algebra II.

Students' Concepts of Pictographs and Bar Graphs

Lionel Pereira-Mendoza

The purpose of this descriptive study was to investigate elementary students' understanding of the information conveyed by pictographs and bar graphs. In particular, the effects of various characteristics such as familiarity with the graph topic, the arrangement of the data, and scale on students' ability to read, interpret and predict from graphs were examined.

A sample of approximately 400 elementary school students participated in this study. Each student was administered a written test, consisting of either bar graphs or pictographs. On the basis of the written responses 100 students given audiotaped interviews to obtain additional information. The responses were then categorized and the major errors discussed in terms of the frame theory model developed by Davis (1987).

While some errors were classified as reading-language and computational errors, the majority were graph-based errors. General categories of errors identified included those derived from data arrangement, topic and scale problems.

Overall students had little difficulty reading graphs or interpreting graphs. The major difficulties occurred when trying to predict from the graphs. Specific problems included making inappropriate predictions and inappropriate use of students preconceived knowledge of the topic. For example, in a non-patterned bar graph dealing with production data, students would tend to impose patterns where none existed. In a pictograph concerned with how children travel to school, students tended to impose preconceived beliefs about travel, even when this apparently contradicted the graphical data.

The paper will expand of these difficulties and discuss them using frame theory. In addition, the paper will address the implications of these difficulties for the instructional practices.

Contributing Factors to the Decline of Blacks in Mathematics

Carolyn L. Pinchback and Jesse Rancifer

Over the past twenty years, the experiences of Black students in meaningful studies in mathematics have become an interest to many educators and researchers. Though some attempts at the secondary level have been made to increase the number of required units in mathematics studies for graduation, Black students are significantly underrepresented in upper level mathematics courses. These mathematics courses have been found to have a significant impact on a student's academic preparation for college, vocational school, and other achievements necessary for career success.

The problem to be studied in this research will identify impediments in mathematics that tend to lower student achievement and prevent the selection of appropriate courses for career enhancement.

Subjects for this study will be students attending selected, historically Black colleges and urban elementary and secondary schools. An opinionnaire, which will be designed by the investigators, will be used to collect the data for the study. It will include some of the unresolved issues from previous researchers. The data will be interpreted by using frequencies and other appropriate statistical tools and inferences will be drawn from a thorough analysis of the results.

The Role of Language in the Construction of Symmetry Knowledge

Vicki Schell

The purpose of this study was to examine the effects of language upon students' schemas, in particular in relation to their conceptualizations of symmetry. At the 1991 RCDPM Annual Conference, we presented results of studying students' conceptualizations of symmetry. These results were specifically described in terms of the van Hiele model of geometric thinking. The awareness of how language facilitates or impedes construction of knowledge would have profound significance on the way we teach.

Students in fourth and seventh grades were interviewed about their understandings of symmetry. One striking result of these interviews was the effects of language upon their schemas. Sometimes students had a more global definition of a term than mathematicians: students called diagonal any slanted line; this interpretation naturally impacted their responses to questions such as "Is the diagonal of a square always a line of symmetry?" Even had a multitude of meanings: "even sides" indicated either that the sides were the same length, or that they were perpendicular to the base (the sides of triangles, including isosceles triangles, were identified as "not even"). At other times, students had a more localized definition than mathematicians: a rectangle was a four-sided plane figure with four right angles, and "two sides are equal, the other two sides are equal, but different from the first two."

There are two sets of the English language utilized in the mathematics classroom: "mathematical English" and "everyday English." There is a non-empty intersection between the two sets, but the words are used differently in the two sets. Teachers have an implicit understanding of this, but students often do not. The results of this study indicate that we need to attend to this difference.

An Intervention to Renew Elementary Mathematics

Mary Ellen Schmidt

A two-week, six-graduate credit hour intervention program is planned for Summer 1991 for 30 (K-8) elementary teachers, who are willing to make a long-term commitment to renewal in elementary mathematics. Teachers will develop their mathematics and pedagogical skills during the summer institute and implement plans in the classroom; they will receive ongoing support during the school year as they implement change in their mathematics program by attending regularly scheduled seminars, and through classroom observations by University staff. As further funding is obtained, these teachers will continue on a 4 year track preparing them as "Elementary Mathematics Specialists" to serve the members of the recently formed North Central Ohio Consortium to Improve mathematics and Science Teaching. During the first year they will be required to share inservice programs reflecting their philosophy of mathematics with other teachers in the North Central Ohio region.

During this summers program, participants will attend sessions team taught by members of the mathematics education and mathematics faculty. The sessions will focus on active participation in problem solving within certain strands of the elementary mathematics curriculum (other strands will be focused on in following years). Teachers will be expected to prepare lessons/units to be implemented in their classroom this school year, as well as inservice programs for other teachers. During the school year, teachers will implement the lessons/units in their classroom, attend 6 follow-up seminars, and present a minimum of 3 inservice programs.

Data will be collected documenting the teachers' development in knowledge, skills, and values. Of particular interest, will be information concerning their common misconceptions, and experiences to help them to construct clearly formed mathematical concepts.

This project is innovative because it will operate under the umbrella of the recently formed North Central Ohio Consortium For the Improvement of Mathematics and Science Teaching. The mission of the consortium will put the university in the center of the regional area, and strengthen the bond between and among the university, schools, and businesses. Bonds with schools will provide pertinent information allowing continual assessment and evaluation of programs in elementary mathematics.

Connecting Mathematical Knowledge

Jean Schmittau

The development of knowledge connections, a major theme of the 1989 NCTM Curriculum and Evaluation Standards, presents many challenges to the mathematics education community. Concepts are developmental by nature; they cannot be taught incrementally via a "skills" model developed with the aid of task analysis. Because of their developmental nature, it is imperative that concepts which may not be encountered until high school influence the design of the curriculum from the earliest years of schooling.

Results of cross cultural studies done in the U.S. and the Soviet Union (where conceptual analysis has figured prominently in the design of the curriculum) will be used to present a model of curriculum design which focuses on conceptual connections. We hope thereby to provide some direction in addressing such questions as the following:

> Can we teach addition in such a way that children's action schemas for "putting together" will connect with the formal operational algorithm a/b + c/d - (ad + bc) /bd?
>
> Can we connect children's ideas of what it means to "divide" with "invert and multiply?"
>
> Can we teach multiplication so that the essential notion of "change of unit" identified by Kolmolgorov connects with the formation of area from the product of length and width?
>
> Can we connect conceptual subdomains such as multiplication of polynomials with multiplication in our base ten positional system?

The presenter's cross cultural studies with U.S. and Soviet students will reference the presentation, which will address these and other questions, and (it is hoped) demonstrate that this task is not only possible, but essential if mathematics is to have meaning for those who study.

Depending on the point of view one wishes to adopt, this is either a thematic presentation with research underpinnings, or a research presentation whose results along the lines of a single theme are presented and explored. I have chosen the first label, but the second fits equally well.

FOCUS ON LEARNING PROBLEMS IN MATHEMATICS: A FORUM FOR PROSPECTIVE JOURNAL WIRTERS AND REVIEWERS

Jean Schmittau

As the editor of the RCDPM journal, Focus on Learning Problems in Mathematics, I found at the 1991 annual meeting in Pomona, that I had to sacrifice many of the paper presentations to meet with those interested in authoring, developing special issues, etc. This occurred in spite of devoting a number of luncheons, "breaks," etc., to this purpose. In spite of this, I feel strongly that I probably missed because of unavailability, some people who wanted to talk with me about such things.

Hence, I would like to try formalizing this process a bit, by providing a combination presentation/exchange of ideas which would:

a) provide an update of directions taken as a result of joint meetings of the publisher (Mahesh Sharma of CT/LM), the chair of the editorial board (Bob Underhill), and myself, for continuing improvements in the journal.

b) inform prospective authors, reviewers, and interested other, of Focus guidelines, what we look for in a manuscript, etc., assist them with the development of ideas, and discuss their work.

c) open the floor to an exchange of ideas for continuing improvement, publicity, etc.

I believe that editors have a responsibility to authors regardless of whether their works are accepted for publication or not, and this is an attempt to assist them in my role as editor.

I would propose the following "description" for the meeting, which might appear either at the conference itself or in advance of it through the RCDPM Newsletter:

Interested in having your RCDPM paper considered for publication in FOCUS? In reviewing for the journal? Do you have an idea for an article or a special issue which you would like to explore further? Come and meet with the editor for an exchange of ideas and a report on continuing improvements for the RCDPM journal, Focus on Learning Problems in Mathematics.

The Diagnostic Prescriptive Mathematics Workshop in PA

G. Bradley Seager, Jr.

In 1987 the Pennsylvania Department of Education supported the formation of a Diagnostic Prescriptive Mathematics Consortium with representation from school districts, intermediate units, universities, and the Department. The Advisory Board of the consortium has guided the development and refinement of videotapes, teachers' and instructors' manuals for the workshops, and research associated with the workshops. The research consists of evaluative surveys and case studies and will be described in the presentation.

Each year, beginning in 1987, members of the Advisory Board have conducted one or two five-day workshops at different universities in Pennsylvania. These workshops have been designed for teachers and other educators to familiarize them with the procedures, rationale, and structure of diagnostic and prescriptive teaching of mathematics. The workshops also contain a component for those who want to prepare themselves to offer workshops in DP Math.

Children participate in the workshops during the afternoon and proceed through the four levels of diagnosis: (1) the Survey Test; (2) the Single Concept Test; (3) the Diagnostic Interview; and (4) Prescription and Ongoing Diagnosis. In 1991 the workshops focused on implementation of the NCTM Standards in the diagnostic prescriptive mathematics program. The workshops stress the use of concrete materials, problem-solving strategies, and cooperative learning in small groups.

If this proposal is accepted for a thematic presentation, illustrations of the four levels of diagnosis will be provided. If it is accepted as a workshop, participants will have experience in simulations of these levels.

Although the original emphasis in developing the workshops was on teaching strategies and techniques for work with low achievers, the scope of the workshops has been broadened to include strategies and techniques for working with children of all abilities.

Workshops using the same instructors' and teachers' manuals are offered throughout the year as requested by school districts and intermediate units. University courses incorporate many of the strategies and techniques developed and demonstrated in the workshops when these courses are taught by members of the Advisory Board and others with experience in the workshops.

School/University Partnership Activities to Assist Preservice Teachers in Integrating Technology in the Classroom

Roslynn (Roz) Seidenstein

In some university courses for preservice teachers, professors are excellent role models for the integration of technology into the classroom. Students may be involved in activities such as evaluating mathematics software programs, using databases and spreadsheets, etc. Further, they may discuss issues about the integration of all of these in their classrooms. However, in other courses, the students may not be provided with such positive experiences.

When these students enter their student teaching experience, they are placed in a variety of classroom situation, from those where teachers use a great deal of technology to those where little is used. (In California, there are huge numbers of teachers in the classroom working with "emergency credential," meaning that they are full time, salaried teachers who request university supervision while on the job.)

We have a professional responsibility to provide opportunities for our students. What skills and experience do our students need? What kind of activities can we provide to meet these needs?

In this session we will share some of the experiences that we've been trying with our students. These include:

- Participating in field trips to model classrooms, to see teachers moving from the use of manipulative materials to online activities
- Planning and implementing, and receiving feedback on technology-intensive activities while student teaching (computer, laser discs, Prodigy, etc.)
- Conferencing with master teacher and university supervisor by Email
- Sharing teaching ideas, lesson plans, and discussions online with supervisor and colleagues (use of FrEdMail)

We hope to provide a forum for discussion with those in the audience.

A Whole Language Approach to Mathematics

Helene J. Silverman

This workshop focuses on the role of language in building understanding of mathematics. A model for constructing meaning will be demonstrated through several tasks designed for use with low achievers in urban elementary schools. As participants engage in the activities, they will be asked to consider how they are using listening, speaking, reading and writing as they interact in cooperative groups and evaluate group and individual performance.

Using Life Histories
as a
Tool for Assessing and Examining Mathematical Attitudes
and the NCTM *Standards*

Lyn Taylor, University of Colorado-Denver

In this presentation I will demonstrate how life histories can be used to assess and study mathematical attitudes and how these relate to the NCTM *Standards (1989).* This research has implications for educators (elementary, secondary, and adult) as well as researchers.

Vygotsky's (1930-4/1978) social interactionist theories and mathematical attitude research, especially Fennema (1989) and Reyes (1984), provide the conceptual framework for this work. This framework and the analysis of mathematical life histories guided the creation of my model of mathematical attitude development (see Figure 1). Directly or tangentially, the framework and the analysis support the idea that students' attitudes develop when they interact with other persons and their environment. This constructivist view also appears to be part of the framework supporting the *Standards* (1989).

Vygotskian conceptual framework. Vygotsky's emphasis on how culture influences learning provides a broad conceptual framework that can take us beyond a strictly cognitive focus and challenge us to examine the learning and teaching of mathematics in the context of cognitive, affective and social dimensions. His Zone of Proximal Development (ZPD), the region between a person's current and potential achievement, is very helpful in gleaning relevant information from life histories.

Attitude. I see attitude as a "way of thinking, feeling and behaving." This broad multidimensional definition goes beyond most definitions and suggests affect is only one part of attitude. When attitude is viewed simply as liking or disliking, the cognitive and behavioral components of attitude are frequently overlooked. The formation of an attitude is a complex process involving the interaction among many factors such as family, socialization, schooling experiences, and relationships with mentors (see Taylor, 1988 & 1990 for further discussion).

Life histories and *the Standards*. My work researching the mathematical life histories of twelve outstanding teachers, I believe illuminates the essence of *the Standards*, as well as the context in which teachers apply *the Standards*. I found many of the goals and specific areas for increased and decreased attention recommended in *the Standards* to be complementary with events in the participants' lives and with Vygotsky's theories.

My model of attitude development emerged from the life history research and is, I believe, very much in line with the five essential goals of *the Standards*. These goals assume that students' should not only be able to solve problems and reason mathematically, but also to become confident and value mathematics, communicate it effectively, make connections and to become aware of how mathematics has impacted their lives. I believe these goals illuminate the importance of and the connections among thoughts, feelings and behaviors (the components of attitude). Therefore, the goals of *the Standards* suggest we be concerned with attitudes.

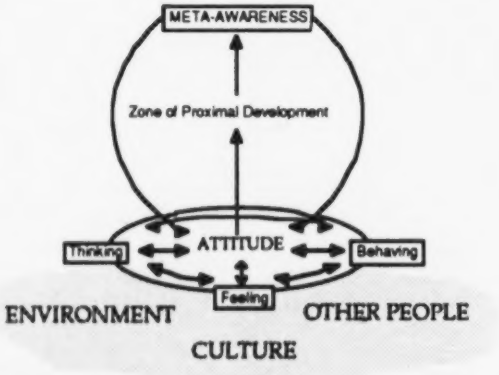

Figure I - A model of mathematical attitude development depicting the components of attitude, bridging the ZPD to meta-awareness and factors influencing this process.

Conclusion. In this session it will become clear how the study of mathematical life histories can be used as a tool to explore mathematical attitudes and the *Standards*. I will illustrate how teachers can use this methodology as an alternative classroom assessment strategy.

Teaching the Language and Structure of Mathematics to Non-Mathematically Oriented College Students

Anne R. Teppo, Bozeman, MT
Warren W. Esty, Montana State University

This paper discusses an innovative class offered for the last four years at Montana State University that focuses on the language, logic, and structure of mathematics. The class has been found to be highly successful for teaching mathematical concepts to non-mathematically oriented, liberal arts students and for improving their attitudes towards and valuations of mathematics. The course emphasizes language skills such as the ability to read mathematics with comprehension, to express mathematical thoughts clearly both orally and in writing, to reason clearly, and to recognize and employ common patterns of mathematical thought. The course culminates in chapters covering the logic of algebraic methods and proofs.

The specific nature of the course content and arrangement of topics play a major role in making the course what we term open-entry, learnable, and meaningful. Information from a qualitative study conducted during the Fall 1990 course provides documentation of students' conceptual attainment and the positive shifts that occurred in the attitudes of this class of predominately math-anxious students. This course can be used as an effective learning experience for students who have failed to develop appropriate underlying concepts of algebraic. The fact that the topics covered in this course are fundamental to mathematics, but not readily available elsewhere, makes this material also appropriate for an advanced course in mathematics education.

"Project UP with Mathematics: A Collaborative Math Clinic Between a University and Local School System"

Karen A. Verbeke

This presentation will focus on the development and implementation of a mathematics clinic, shared collaboratively by a university (University of Maryland Eastern Shore) and a local school system (Somerset County Public Schools - Princess Anne Elementary School). Both the university and local school system are located in a rural area, with a high minority population. The mathematics clinic, part of the collaborative project (Project UP with Mathematics) involved a variety of participants, including a university professor, undergraduate and graduate students from the university, classroom teachers (second grade team), second graders, and local administrators (principal and curriculum director). The overall project included several components. Specifically:

1) instruction of mathematics content and methods course taught on-site in local school classroom
2) supervised mathematics clinical experience in local school;
3) on-site demonstration lessons for both university student clinicians and classroom teachers, and
4) observation opportunities for university students.

This presentation will highlight the overall organization of the clinic, the role of the participants, its impact on the clinicians, students and teachers, and the nature of the experiences and activities in the clinic. The clinical experience was evaluated using surveys, interviews, session reports, videotaping, peer teaching evaluations, and case studies. Specific issues were addressed in the evaluation:

1) self-confidence of clinician
2) knowledge of clinician about mathematics content
3) use of materials by clinician
4) approach to behavior management by clinician
5) new insights (i.e., mathematical and pedagogical) by clinician
6) confidence in mathematics by students in clinic
7) competence in mathematics by students in clinic.

In addition to these findings, future directions for expansion and modification of this program model will be included.

STIMULATE MATHEMATICAL THINKING IN SECONDARY STUDENTS: A DYNAMIC APPROACH

FLORENCE VOLD AND ELEANOR HILLEGEIST

We have completely changed not only the approach we are using to teaching mathematics but have also come a great distance in figuring out how to assess mathematical behavior - as we define it! We are involved in teaching mathematics through a dynamic problem solving approach to instruction that engages students in presenting their mathematical ideas in lively discussions and in written form. By comparing both we can begin to see into the window of their thinking. Then we can begin to help them fine tune their own thinking process.

We know that our students are capable of thinking and therefore we are giving them interesting questions to think about and are challenging them to explain to each other and to us, how it is they are thinking about these questions - and what other questions do our questions bring to mind. All of this is happening through a dynamic process of classroom discussion followed by question (problem) resolutions presented in written language format. We read and comment on their written ideas for organization and mathematical content - not for perfect English presentation.

We are developing a way of teaching mathematical skills and concepts in geometry and algebra through a problem solving process that requires the students to think about questions like mathematicians do. What does this mean for us? Well, right now it means using two processes we think are used in resolving mathematical questions. These processes are presented under many names in the problem solving literature. We are most comfortable with calling them search and generalize processes and we think these processes are well suited for mathematical thinking. So far, we think of SEARCH as the part of the process in which we look for relevant examples or diagrams to represent the questions. The GENERALIZE part of the process requires students to search for patterns in the examples they generate or diagrams they create to represent the information in the questions we or they generate.

We are becoming keenly aware of three factors that influence the development of mathematical reasoning: (How are we addressing these?) 1) competence in searching and generalizing; 2) confidence in handling emotional and psychological states when engaged in mathematical work and turning these states to advantage and 3) understanding of the content of mathematics and the areas in which it is applied. We would like to facilitate the development of all three areas in our students, even though we have them for only two semesters. We feel a need for developing better assessment methods that will help us evaluate mathematical thinking no matter what stage of development it is in.

We and our students are finding that mathematical thinking can be stimulated and supported in an atmosphere of questioning, challenging conjectures and reflecting. It can be provoked by experiencing contradiction, tension and surprise. Mathematical thinking is something we all can do and helps in understanding ourselves and the world.

The Developmental Logic K-12: A Missing Link to Learning

George A. Wyer

Logic has emerged as a clear missing link to learning. Without Logic as a meta-system for creating, extending, and applying systems of ideas, we risk being overwhelmed by the knowledge explosion that is now dubbed the "Information Age." It is important to be cognizant of student problems working with any system specific language (vocabulary), and accessing or applying the rules (axioms and theorems) inherent in a system of ideas. This is not less true of Logic. Logic should possess a universally agreed upon and efficient symbolism, be developed axiomatically as a well-defined system of ideas and be appropriately applied across the disciplines.

In 1968, while teaching Logic and Geometry at Rhode Island Junior College, I wrote a first approximation to such a development of Logic which, with its refinements over the years, has evolved into an effective two week curriculum unit which I have since revised and used successfully in a variety of contexts. These include teaching Logic and Geometry to prospective elementary and secondary school teachers as well as teaching high school Geometry.

Interestingly, three points of the "nine-point program for college-capable students" proposed in 1959 by The Commission on Mathematics of the College Entrance Examination Board were Logic related and never fully realized. These were: "2) understand the nature and role of deductive reasoning, 3) appreciate mathematical structure (properties) and 4) use unifying, set-theoretic ideas" (32nd NCTM Yearbook, 1970 p. 260). I wonder if a unifying predictive theory on the teaching and learning of mathematics or, perhaps, performance based testing would have made a difference? Thirty years later...

The widely distributed Curriculum and Evaluation Standards for School Mathematics, published in March of 1989 by the National Council of Teachers of Mathematics, calls for increased attention to "extending logical (deductive) reasoning skills" so that all students can, for example, "follow logical arguments, judge the validity of arguments and construct simple valid arguments" (p. 143). It also noted "that potential for transfer between mathematical reasoning and the logic needed to resolve issues in everyday life can be enhanced by explicitly subjecting assertions about daily affairs to analysis in terms of the underlying principles of reasoning" (p. 145).

I could not agree more. But here the Standards falls short. Not only does it suggest Euclidean Geometry as a complete axiomatic system receive decreased attention, but no mention is made of what the "underlying principles" are or of their development. Thus, there is still no direction for the inclusion of Logic in the pre-college curriculum, and no indication any is forthcoming.

The "underlying principles" of reasoning, referenced but not stated in the NCYM Standards (1989), will be described as part of a classroom-tested, axiomatic development of Logic appropriate for grade 10 or above. The prerequisite development in grades K-8 of simple arguments, language connectives and set theory will be addressed along with suggestions for needed clinical intervention research.

FOSTERING PROFESSIONAL GROWTH: A CRITERION-REFERENCED FORMAT FOR STUDENT TEACHERS IN MATHEMATICS

Alan Zollman

Society, and educators at all levels, are currently demanding accountability and instructional quality in mathematics teaching. One method to respond to this demand is teacher assessment. Especially for the student teacher, assessment instruments are frequently very nebulous. Neither the format nor the process is precise or developmental (Smith & Stevens, 1984). One major goal for assessment of student teachers is to nurture the inexperienced teacher. In middle and high school mathematics teaching, generic formats tell little about the specific teaching behaviors for teaching school mathematics (NCTM, 1990). A subject specific assessment for school mathematics would be useful. The purpose of this paper is to provide a paradigm for the development of a criterion-referenced format to meet this exigency.

To assess and nurture mathematics student teachers, one approach is the use of a criterion-referenced format instrument by the cooperating classroom teacher. A criterion-referenced format serves five functions as it: (1) provides a reference; (2) sets expectations; (3) guides the cooperating teacher in systemic observation; (4) encourages diagnostic feedback for improvement; and (5) provides a continuum as a base of dialogue among the student teacher, the cooperating teacher, and the university supervisor. In criterion-referenced measures a student teacher's performance is judged in terms of behaviorally stated, observable tasks which sample the critical competencies of teaching the subject (Smith & Stevens, 1984). In light of the National Council of Teachers of Mathematics Curriculum and Evaluation Standards for School Mathematics and Professional Standards for Teaching Mathematics such a criterion-referenced evaluation format is beneficial (NCTM, 1989; NCTM, 1991).

A criterion-referenced format is beneficial as it encourages professional growth in the student teacher. If the format is behaviorally stated, the feedback can be diagnostic and therefore useful for professional development. A criterion-referenced format is useful as a guide as it promotes a nurturing dialogue among the student teacher, the cooperating teacher, and the university supervisor. Finally, a criterion-referenced format also can reduce the time for complex reporting and analysis. Thus, it is likely to be used correctly in terms of its objectives.

Specifically in middle and high school mathematics teaching, the format design also must deal with three areas for evaluation: knowledge of teaching pedagogy, knowledge of mathematics, and knowledge of school mathematics. Teaching pedagogy entails generic teacher competencies. Knowledge of school mathematics differs from knowledge of mathematics as it requires the skills of communicating school mathematics to one who does not understand. An example of teaching pedagogy knowledge would be questioning strategies; an example of knowledge of mathematics would be solving simultaneous linear equations; and an example of knowledge of school mathematics would be demonstrating factoring trinomials using rectangular areas while relating this to factoring whole numbers.

In setting up the criterion-referenced instrument, there should be a division between a behavior occurring and the quality of the behavior for each criterion. To this purpose, the criterion-referenced format could ask for: (1) the frequency of the teaching behavior, (2) the quality of the behavior, and (3) supporting evidence, comments, and/or suggestions on each criterion.

Evaluation of teaching is a process of gathering information and providing a basis for interpreting the information about the very complex task of teaching mathematics. The process should be longitudinally used throughout the student teaching experience, a formative not summative evaluation. The goal of evaluation of student teaching is to assist student teachers to improve their teaching and develop professionally. A criterion-referenced format can be a valuable tool in this respect as it naturally establishes a line of professional communications among the student teacher, the supervising teacher, and the university coordinator.

As with any evaluation instrument, a criterion-referenced form should only be one aspect of the evaluation process. The importance of formative comments by the supervising teacher should not be disregarded. A criterion-referenced format is in no way comprehensive, as no practical instrument could be, but it can be a basis of communicating valued competencies in the teaching of mathematics.